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A New Modified Sumudu Transform Called Raj Transform to Solve Differential Equations and Problems in Engineering and Science

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ABSTRACT: Solving differential equations and finding solution of homogeneous and non-homogeneous differential equations are not very easy one in some critical situations. To use certain method to find every different equations. Most of the engineering problems were converted to be in differential equations and solve by using Z-transform, Laplace transform, Fourier transform etc., particularly in continuous case Laplace transform are using but some complex differential equations solved by Laplace transform is difficult. In this paper a new modified Sumudu transform introduced called Raj transform to solve differential equations and fuzzy differential in engineering problems. This transform can solve differential equations and fuzzy differential equations especially boundary and initial conditions problems. Important properties with proof of the new transform also derived. Dualities between new integral transform and other integral transform also provided. Finally to understand this new integral transform, two numerical examples also given with graphical explanation in the end of the paper. Using Raj transform is easy to solve complex differential equations in both homogeneous and non-homogeneous higher order differential equations. This method is very interesting to solve different type of difficult problems in both engineering and real life.

Keywords: Differential equation, Modified Sumudu, New Integral Transform, Raj Transform, Sumudu Transform.

I. INTRODUCTION

Many of the real life problems, science problems and engineering problems were solved by integral transform. Integral transform plays very important usage to solve differential and partial differential equation. Most of the integral transform is mapped into t domain into s domain by integral transform. Very famous known transform Fourier, z-transform and Laplace transform. Mostly z transform is used to solve discrete case of problems and Laplace is used to solve continuous case problems. in these transform the changeable variable in s domain treated as dummies and also physical significance is also not questioned. Jena and Mohanty (2019) worked on ODE using numerical technique [1]. Watugala (1993) introduce integral transform named Sumudu in 1993 to solve control engineering problems [2]. Many researcher are discussed with different transform like Mohgoub transform, Aboodh transform, Kamal transform, Elzaki transform, Mohand transform and Sawi transform to solved an engineering mathematical problems [3-7]. Aggarwal and Chaudhary (2019) worked on comparative study on Mohand transform with Laplace to solve differential equation [8]. Recently many research scholars used different type of integral transform to solve different type of problems in both engineering and real life [9-22]. Here the new modified Sumudu transform of *t* domain is used for dividing *u* domain. It owns so many interesting properties; new integral transform is related with other transform like Laplace, Sumudu, etc. differential equation are converted into fuzzy differential equation and used new integral transform to solved the equation by using initial condition as a fuzzy parameter (numbers). Allahviranloo *et al.*, worked on differential equations in fuzzy environment [23-26]. Melliani *et al.*, (2015) also solved differential equation in fuzzy environment [27]. Rajkumar and Jesuraj modeled a real life problem into differential equation and its solved by using fuzzy numbers like triangle, nonagonal [28-30]. Recently up to 2019 many others used to solve differential and partial differential equation by the known transform but few problems are not able to solve in same transform, this transform used to solve many problems in complex domain.

Final section of the paper given few numerical examples to understand the new method. Application of this new transform can used to solve mathematical problems, stress analysis, signal processing, civil engineering, control system mechanics, heat conduction, electricity, deflection of beams, etc.

II. MATERIALS AND METHODS

A. Laplace transform

The laplace transform Z of a function $f(\varsigma)$ for ς is greater than zero given by

$$Z(f(\varsigma)) = \int_0^\infty f(\varsigma)e^{-s\varsigma}d\varsigma$$

B. Sumudu transform

The Sunudu transform Z of a function $f(\varsigma)$ for ς is greater than or equal to zero given by

$$Z(f(\varsigma))=\int_0^\infty f(s\varsigma)e^{-\varsigma}d\varsigma \ .(0<k_1\leq\xi\leq k_2)$$

C. Mahgoub transform

The Mahgoub transform Z of function f(ς) for ς is greater than or equal to zero given by

$$Z(f(\varsigma))=s\int_0^\infty f(\varsigma)e^{-s\varsigma}d\varsigma \ .(0<k_1\leq\xi\leq k_2)$$

D. Elzaki transform

The Mahgoub transform Z of a function f(ς) for ς is greater than or equal to zero given by

$$Z(f(\varsigma))=s\int_0^\infty f(\varsigma)e^{-\frac{\varsigma}{s}}d\varsigma \ .(0<k_1\leq\xi\leq k_2)$$

E. Aboodh transform

The Aboodh transform Z of a function f(ς) for ς is greater than or equal to zero given by

$$Z(f(\varsigma))=\frac{1}{s}\int_0^\infty f(\varsigma)e^{-s\varsigma}d\varsigma \ .(0<k_1\leq\xi\leq k_2)$$

F. New Integral Transform Modified Sumudu Called Raj Transform

The modified Sunudu transform Z of a function f(ς) for ς is greater than or equal to zero given by

$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$ where ξ is in between zero and infinity $(0<k_1\leq\xi\leq k_2)$,here k_1 and k_2 are either finite or infinite

III. RESULTS AND DISCUSSION

A New Modified Sumudu Transform (Raj Transform) for Basic Function

A. Let $f(\varsigma)=1,$ by using transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma,\ Z(1)=\int_0^\infty f(\frac{1}{s})e^{-\varsigma}d\varsigma=\frac{1}{s}\int_0^\infty e^{-\varsigma}d\varsigma=\frac{1}{s}$$

B. Let $f(\varsigma)=e^{a\varsigma},$ by using transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$Z(e^{a\varsigma})=\int_0^\infty e^{\frac{a\varsigma}{s}}e^{-\varsigma}d\varsigma$$

$$=\int_0^\infty e^{-\varsigma+\frac{a\varsigma}{s}}d\varsigma$$

$$=\frac{1}{1-\frac{a}{s}}$$

$$Z(e^{a\varsigma})=\frac{s}{s-a}$$

C. Let $f(\varsigma)=\varsigma,$ by using transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$Z(\varsigma)=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$=\frac{1}{s}\int_0^\infty \varsigma e^{-\varsigma}d\varsigma$$

$$=\frac{1}{s}$$

D. Let $f(\varsigma)=\varsigma^n,$ by using transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$Z(\varsigma^n)=\int_0^\infty f(\frac{\varsigma^n}{s})e^{-\varsigma}d\varsigma$$

$$=\frac{1}{s}\int_0^\infty \varsigma^n e^{-\varsigma}d\varsigma$$

$$Z(\varsigma^n)=\frac{n!}{s^n}$$

E. Let $f(\varsigma)=\sin(a\varsigma),$ by using transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$Z(\sin(a\varsigma))=\int_0^\infty \sin(\frac{a\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$=\frac{as}{a^2+s^2}$$

$$Z(\sin(a\varsigma))=\frac{as}{a^2+s^2}$$

F. Let $f(\varsigma)=\cos(a\varsigma),$ by using transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$Z(\cos(a\varsigma))=\int_0^\infty \cos(\frac{a\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$=\frac{s^2}{a^2+s^2}$$

$$Z(\sin(a\varsigma))=\frac{s^2}{a^2+s^2}$$

G. Let $f(\varsigma)=\sinh(a\varsigma),$ by using transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$Z(\sinh(a\varsigma))=\int_0^\infty \sinh(\frac{a\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$=\frac{as}{s^2-a^2}$$

$$Z(\sin(a\varsigma))=\frac{as}{s^2-a^2}$$

H. Let $f(\varsigma)=\cosh(a\varsigma)$, by using transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$Z(\cosh(a\varsigma))=\int_0^\infty \cosh(\frac{a\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$=\frac{s^2}{s^2-a^2}$$

$$Z(\sin(a\varsigma))=\frac{s^2}{s^2-a^2}$$

Relation between a New Integral Transform and Other Integral Transform

A. Laplace and Raj transform

Theorem: 1

If laplace and raj transform of $Z(f(\varsigma))$ are L(ς) and R(ς)

$$L(\varsigma)=\frac{1}{S}R(\varsigma) \text{ and}$$

$$R(\varsigma)=sL(\varsigma)$$

Proof :from the definition of laplace transform we have

$$Z(f(\varsigma))=\int_0^\infty f(\varsigma)e^{-s\varsigma}d\varsigma$$

Put $s\varsigma=p,\ d\varsigma=\frac{dp}{s}$ in the above equation

$$Z(f(\varsigma))=\frac{1}{s}\int_0^\infty f(\frac{p}{s})e^{-p}dp$$

$$=\frac{1}{s}R(Z(f(\varsigma))$$

Similarly we have raj transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

Put $\frac{\varsigma}{s}=p,\ d\varsigma=sdp$ in the above equation

$$Z(f(\varsigma))=s\int_0^\infty f(p)e^{-sp}dp$$

$$=sL(Z(f(\varsigma))$$

B.Sumudu and Raj transform

Theorem: 2

If Sumudu and raj transform of $Z(f(\varsigma))$ are S(ς) and R(ς)

$$S(\varsigma)=\frac{1}{S}R(\varsigma) \text{ and}$$

$$R(\varsigma)=sS(\varsigma)$$

Proof : from the definition of Sumudu transform we have

$$Z(f(\varsigma))=\int_0^\infty f(s\varsigma)e^{-\varsigma}d\varsigma$$

Put $s\varsigma=p,\ d\varsigma=\frac{dp}{s}$ in the above equation

$$Z(f(\varsigma))=\frac{1}{s}\int_0^\infty f(\frac{p}{s})e^{-p}dp$$

$$=\frac{1}{s}R(Z(f(\varsigma))$$

Similarly we have raj transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

Put $\frac{\varsigma}{s}=p,\ d\varsigma=sdp$ in the above equation

$$Z(f(\varsigma))=s\int_0^\infty f(sp)e^{-p}dp$$

$$=sS(Z(f(\varsigma))$$

C. Mahgoub and Raj transform

Theorem: 3

If Mahgoub and raj transform of $Z(f(\varsigma))$ are M(ς) and R(ς)

$$M(\varsigma)=R(\varsigma) \text{ and}$$

$$R(\varsigma)=M(\varsigma)$$

Proof : from the definition of laplace transform we have

$$Z(f(\varsigma))=s\int_0^\infty f(\varsigma)e^{-s\varsigma}d\varsigma$$

Put $s\varsigma=p,\ d\varsigma=\frac{dp}{s}$ in the above equation

$$Z(f(\varsigma))=\frac{1}{s}\int_0^\infty f(\frac{p}{s})e^{-p}dp$$

$$=R(Z(f(\varsigma))$$

Similarly we have raj transform

$$Z(f(\varsigma))=\int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

Put $\frac{\varsigma}{s}=p,\ d\varsigma=sdp$ in the above equation

$$Z(f(\varsigma)) = s\int_0^\infty f(p)e^{-sp}dp$$
$$= M(Z(f(\varsigma))$$

D. Elzaki and Raj transform

Theorem: 4

If Elzaki and raj transform of $Z(f(\varsigma))$ are E(ς) and R(ς)

$$E(\varsigma) = \frac{1}{S}R(\varsigma) \text{ and}$$

$$R(\varsigma) = s^2E(\varsigma)$$

Proof : from the definition of Elzakitrans form we have

$$Z(f(\varsigma)) = \frac{1}{s}\int_0^\infty f(\varsigma)e^{-s\varsigma}d\varsigma$$

Put $s\varsigma = p,\ d\varsigma = \frac{dp}{s}$ in the above equation

$$Z(f(\varsigma)) = \frac{1}{s}\int_0^\infty f(\frac{p}{s})e^{-p}dp$$
$$= \frac{1}{s}R(Z(f(\varsigma))$$

Similarly we have raj transform

$$Z(f(\varsigma)) = \int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

Put $\frac{\varsigma}{s} = p,\ d\varsigma = sdp$ in the above equation

$$Z(f(\varsigma)) = s\int_0^\infty f(p)e^{-sp}dp$$
$$= s^2E(Z(f(\varsigma))$$

E. Aboodh and raj transform

Theorem: 5

If Aboodh and raj transform of $Z(f(\varsigma))$ are A(ς) and R(ς)

$$L(\varsigma) = \frac{1}{S^2}R(\varsigma) \text{ and}$$

$$R(\varsigma) = s^2A(\varsigma)$$

Proof : from the definition of Aboodh transform we have

$$Z(f(\varsigma)) = \frac{1}{s}\int_0^\infty f(\varsigma)e^{-s\varsigma}d\varsigma \ .(0<k_1\leq \xi \leq k_2)$$

Put $s\varsigma = p,\ d\varsigma = \frac{dp}{s}$ in the above equation

$$Z(f(\varsigma)) = \frac{1}{s}\int_0^\infty f(\frac{p}{s})e^{-p}\frac{dp}{s}$$
$$= \frac{1}{s^2}R(Z(f(\varsigma))$$

Similarly we have raj transform

$$Z(f(\varsigma)) = \int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

Put $\frac{\varsigma}{s} = p,\ d\varsigma = sdp$ in the above equation

$$Z(f(\varsigma)) = s\int_0^\infty f(p)e^{-sp}dp$$

$$= s^2\left(\left[\frac{1}{s}\right]\int_0^\infty f(p)e^{-sp}dp\right) = sA(Z(f(\varsigma))$$

Relation for Finding This New Integral Transform with Other Transforms.

Tabular representation of Raj transform with other integral transform.

Table 1: Laplace and Raj transform.

S.No.	Function $f(\varsigma)$	Laplace transform $L(f(\varsigma))$	Raj transform $R(f(\varsigma))$
1.	1	$\frac{1}{s}$	$\frac{1}{s}$
2.	ς	$\frac{1}{s^2}$	$\frac{1}{s}$
3.	ς^2	$\frac{2!}{s^3}$	$\frac{2}{s^2}$
4.	ς^n	$\frac{n!}{s^{n+1}}$	$\frac{n!}{s^n}$
5.	$e^{a\varsigma}$	$\frac{1}{s-a}$	$\frac{s}{s-a}$
6.	$\sin a\varsigma$	$\frac{a}{a^2+s^2}$	$\frac{as}{a^2+s^2}$
7.	$\cos a\varsigma$	$\frac{s}{a^2+s^2}$	$\frac{s^2}{a^2+s^2}$
8.	$\sinh a\varsigma$	$\frac{a}{s^2-a^2}$	$\frac{as}{s^2-a^2}$
9.	$\cosh a\varsigma$	$\frac{s}{s^2-a^2}$	$\frac{s^2}{s^2-a^2}$

Table 2: Sumudu and Raj transform.

S.No.	Function $f(\varsigma)$	Sumudu transform $S(f(\varsigma))$	Raj transform $R(f(\varsigma))$
1.	1	1	$\frac{1}{s}$
2.	ς	s	$\frac{1}{s}$
3.	ς^2	$2!s^2$	$\frac{2}{s^2}$
4.	ς^n	$n!s^n$	$\frac{n!}{s^n}$
5.	$e^{a\varsigma}$	$\frac{1}{1-as}$	$\frac{s}{s-a}$
6.	$\sin a\varsigma$	$\frac{as}{1+(as)^2}$	$\frac{as}{a^2+s^2}$
7.	$\cos a\varsigma$	$\frac{1}{1+(as)^2}$	$\frac{s^2}{a^2+s^2}$
8.	$\sinh a\varsigma$	$\frac{as}{1-(as)^2}$	$\frac{as}{s^2-a^2}$
9.	$\cosh a\varsigma$	$\frac{1}{1-(as)^2}$	$\frac{s^2}{s^2-a^2}$

Table 3: Mahgoub and Raj transform

S.No.	Function $f(\varsigma)$	Mahgoub transform $M(f(\varsigma))$	Raj transform $R(f(\varsigma))$
1.	1	1	$\frac{1}{s}$
2.	ς	$\frac{1}{s}$	$\frac{1}{s}$
3.	ς^2	$\frac{2!}{s^2}$	$\frac{2}{s^2}$
4.	ς^n	$\frac{n!}{s^n}$	$\frac{n!}{s^n}$
5.	$e^{a\varsigma}$	$\frac{s}{s-a}$	$\frac{s}{s-a}$
6.	$\sin a\varsigma$	$\frac{as}{a^2+s^2}$	$\frac{as}{a^2+s^2}$
7.	$\cos a\varsigma$	$\frac{s^2}{a^2+s^2}$	$\frac{s^2}{a^2+s^2}$
8.	$\sinh a\varsigma$	$\frac{as}{s^2-a^2}$	$\frac{as}{s^2-a^2}$
9.	$\cosh a\varsigma$	$\frac{s^2}{s^2-a^2}$	$\frac{s^2}{s^2-a^2}$

Table 4: Elzaki and Raj transform.

S.No.	Function $f(\varsigma)$	Elzaki transform $E(f(\varsigma))$	Raj transform $R(f(\varsigma))$
1.	1	s^2	$\frac{1}{s}$
2.	ς	s^3	$\frac{1}{s}$
3.	ς^2	$2!s^4$	$\frac{2}{s^2}$
4.	ς^n	$n!s^{n+2}$	$\frac{n!}{s^n}$
5.	$e^{a\varsigma}$	$\frac{s^2}{1-as}$	$\frac{s}{s-a}$
6.	$\sin a\varsigma$	$\frac{as^3}{1+(as)^2}$	$\frac{as}{a^2+s^2}$
7.	$\cos a\varsigma$	$\frac{s^2}{1+(as)^2}$	$\frac{s^2}{a^2+s^2}$
8.	$\sinh a\varsigma$	$\frac{as^3}{1-(as)^2}$	$\frac{as}{s^2-a^2}$
9.	$\cosh a\varsigma$	$\frac{s^2}{1-(as)^2}$	$\frac{s^2}{s^2-a^2}$

Table 5: Aboodh and Raj transform.

S.No.	Function $f(\varsigma)$	Aboodh transform $A(f(\varsigma))$	Raj transform $R(f(\varsigma))$
1.	1	$\frac{1}{s^2}$	$\frac{1}{s}$
2.	ς	$\frac{1}{s^3}$	$\frac{1}{s}$
3.	ς^2	$\frac{2!}{s^4}$	$\frac{2}{s^2}$
4.	ς^n	$\frac{n!}{s^{n+2}}$	$\frac{n!}{s^n}$
5.	$e^{a\varsigma}$	$\frac{1}{s(s-a)}$	$\frac{s}{s-a}$
6.	$\sin a\varsigma$	$\frac{a}{s(a^2+s^2)}$	$\frac{as}{a^2+s^2}$
7.	$\cos a\varsigma$	$\frac{1}{a^2+s^2}$	$\frac{s^2}{a^2+s^2}$
8.	$\sinh a\varsigma$	$\frac{a}{s(s^2-a^2)}$	$\frac{as}{s^2-a^2}$

9.	$\cosh a\varsigma$	$\frac{1}{s^2-a^2}$	$\frac{s^2}{s^2-a^2}$

Raj Transform of Derivatives

A. Theorem:

If $Z(f(\varsigma)) = R(s)$ then $R(f^{/}(\varsigma)) = sR(s) - sf(0)$

Proof:

By the definition of raj transform

$$Z(f(\varsigma)) = \int_0^\infty f(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$Z(f'(\varsigma)) = \int_0^\infty f'(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

Method of integration by parts

$$Z(f'(\varsigma)) = \int_0^\infty f'(\frac{\varsigma}{s})e^{-\varsigma}d\varsigma$$

$$u = e^{-\varsigma}, du = -e^{-\varsigma}d\varsigma$$

$$dv = f'(\frac{\varsigma}{s})d\varsigma$$

$$v = sf(\frac{\varsigma}{s})$$

$$= uv - \int vdu$$

$$= \left[(se^{-\varsigma}f(\frac{\varsigma}{s}))\right]_0^\infty - \int_0^\infty sf(\frac{\varsigma}{s})(-e^{-\varsigma}d\varsigma)$$

$$= sR(s) - sf(0)$$

Hence $R(f^{/}(\varsigma)) = sR(s) - sf(0)$

B. Theorem:

If $Z(f(\varsigma)) = R(s)$ then

$$R(f^{//}(\varsigma)) = s^2R(s) - s^2f(0) - sf^{/}(0)$$

Proof: let $R(f^{//}(\varsigma)) = s[sR(s) - sf(0)] - sf^{/}(0)$

By using theorem 6.1 we get, $R(f^{//}(\varsigma)) = s^2R(s) - s^2f(0) - sf^{/}(0)$ hence proved.

Numerical Application

Example 1: Solve the following first order differential equation by raj transform method $y^{/}(t) - 4y = e^t$ where y(0)=0.

Solution: Given , $y^{/}(t) - 4y = e^t$,y(0)=0

Taking raj transform on both sides we get

$$[sR(s) - sf(0)] - 4R(s) = \frac{s}{s-1}$$

$$R(s) = \frac{s}{(s-1)(s-4)}$$

Using partial fraction and inverse raj transform we get

$$y(t) = \frac{1}{3}\left[4e^{4t} - e^t\right]$$

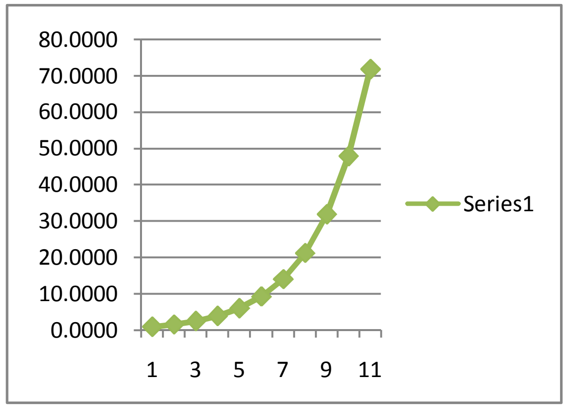

Fig. 1. (when t=0, 0.1, 0.2…1) will get the following graphical representation).

Example 2: Solve the following second order differential equation by raj transform method $y^{//}(t) - y = 1$ where $y_1(0)$=0, y(0)=0.

Solution: Given $y^{//}(t) - y = 1$

where $y_1(0)$=0, y(0)=0.

Using raj transform on both sides

We get

$$s^2R(s) - s^2f(0) - sf^{/}(0) - R(s) = \frac{1}{s}$$

$$R(s)[s^2 - 1] = \frac{1}{s}$$

$$R(s) = \frac{1}{s(s^2-1)}$$

$$= \frac{1}{s(s+1)(s-1)}$$

Using partial fraction and inverse raj transform we get ,

$$y(t) = \frac{1}{2}[e^t + e^{-t} - 2]$$

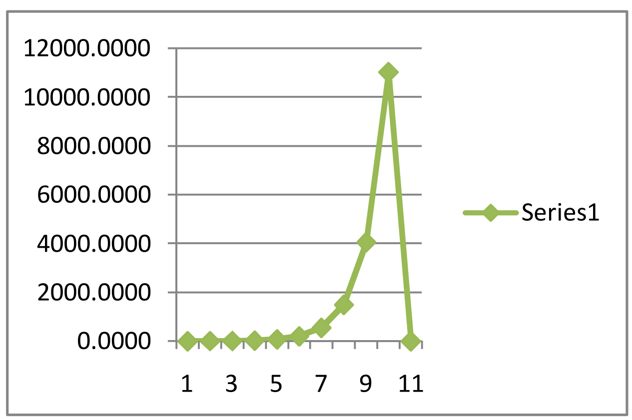

Fig. 2. (when t=1, 2, 3,…10, 1) will get the following graphical representation).

IV. CONCLUSION

The new introduced method is defined and explained the properties through derivations. This transform is related with other integral transform to show the duality between other five transforms .raj transform first and second order derivatives also derived based on the result two numerical examples also solved. Using this transform is very easy to solve differential equation problem or linear and non-liner cases.

V. FUTURE SCOPE

This can help for researcher to solve higher order differential equations, Partial differential, fractional equations and other problems in engineering and real life.

Conflict of Interest. The authors confirm that there are no known conflicts of interest associated with this publication of this paper.

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